

GFD I, Problem Set 5 Solutions, 3/1/2012

1.(a) [5 pts.] In the “hydrostatic, rotating” regime, the only change in the governing equations is that we may drop the vertical acceleration term from Z-MOM, leaving:

$$\text{Z-MOM} \quad 0 = -\frac{1}{\rho_0} p'_z + b$$

(b) [5 pts.] The x-component of the group velocity is given by

$$C_g^x = \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k} \left[f^2 + \left(\frac{Nk}{m} \right)^2 \right]^{1/2} = \frac{1}{2} \frac{1}{\omega} \frac{2N^2 k}{m^2} = \frac{N^2 k}{\omega m^2}$$

(c) [10 pts.] For the “flow over corrugated hills” problem the x-component of the group velocity may be written as:

$$\begin{aligned} C_g^x &= \frac{N^2 k}{\omega m^2} = \frac{N^2}{U} \frac{k}{m^2} \left[\frac{U}{k} \right] \left[\frac{k^2}{k^2} \right] \\ &= U \frac{N^2}{U^2 k^2} \frac{k^2}{m^2} = U \frac{N^2}{\omega^2} \frac{\omega^2 - f^2}{N^2} = U \left(1 - \frac{f^2}{\omega^2} \right) \end{aligned}$$

where we have made use of the relations: $\omega = Uk$ and $k^2/m^2 = (\omega^2 - f^2)/N^2$.

(d) [10 pts.] As the wave frequency approaches the Coriolis frequency, $\omega \rightarrow f$, the x-component of the group velocity goes to zero. For Poincare waves the dispersion relation (Gill 7.3) is: $\omega^2 = f^2 + k^2 c^2$ where $c^2 = gH$ and we have simplified by assuming $l = 0$.

Thus you may show that $k^2 = (\omega^2 - f^2)/c^2$, and in the limit $\omega \rightarrow f$ this shows that

$k \rightarrow 0$. Then, calculating the group velocity:

$$C_g^x = \frac{\partial \omega}{\partial k} = \frac{kc^2}{\omega} \rightarrow 0$$

So in both cases you can no longer propagate energy as the wave frequency approaches the Coriolis frequency. The motion becomes inertial circles, and the horizontal scale becomes very large. Overall we may conclude that Earth’s rotation makes it more difficult for these waves to transmit energy.

2.(a) [5 pts.] In the “non-hydrostatic, non-rotating” regime we may drop both terms containing the Coriolis frequency, leaving:

$$\begin{array}{ll} \text{X-MOM} & u_t = -\frac{1}{\rho_0} p'_x \\ \text{Y-MOM} & v_t = 0 \end{array}$$

As long as there was no v -velocity at the start, you will always have $v = 0$.

(b) [15 pts.] Using DENS, we find directly that

$$b = -N^2 \int w dt = \frac{-N^2}{-\omega} \left[-\delta k U \sin(kx + mz - \omega t) \right] = -N^2 \delta \sin(kx + mz - \omega t)$$

This makes sense physically near the topography because it predicts, e.g., b will have its maximum negative value at the peaks of hills. This is because the deformations of the isopycnals are following the topography, and when you push isopycnals *up* the buoyancy is negative (i.e. a *downward* buoyancy force). Furthermore, the magnitude of the buoyancy perturbation at the hill is just what you would expect from pushing the background stratification up a distance δ .